I–Sem/COMMON/2019(S)(New)

ENGG. MATH - I

(Theory : 3)

Full Marks : 80

Time : 3 hours

Answer any **five** questions including **Q. Nos. 1** and **2**

Figures in the right-hand margin indicate marks

1. Answer *all* : 2 × 10

(a) Solve $\begin{vmatrix} 4 & x+1 \\ 3 & x \end{vmatrix} = 5$.

(b) Form a 2×2 matrix with elements,

a_{ij} if $a_{ij} = i \cdot j$

(c) Find the value of $\sin^2 24^\circ - \sin^2 6^\circ$.

(d) Find the distance between the lines $3x - 1 = 0$ and $x + 3 = 0$.

(e) Find the angle between the lines

$x = 2$ and $x - \sqrt{3} \cdot y + 1 = 0$

(f) Find the equation of the circle whose centre is (2, –3) and radius is 4.

(g) Find the direction cosines of the normal to the plane $x + y + 1 = 0$.

(h) Determine the centre and radius of the sphere $x^2 + y^2 + z^2 - 4x + 6y - 8z + 1 = 0$.

(i) Determine the value of k such that the planes $x + 3y + kz = 5$ and $kx + y + 2z = 0$ are perpendicular to each other.

(j) Find the image of the point (–6, 2, –3) w.r.t yz-plane.

2. Answer any *six* : 5 × 6

(a) Find the maximum and minimum value of $8\sin x - 15\cos x - 1$.

(b) Find the equation of the circle, the end points of a diameter being (–4, 3), (2, –2).

(c) Solve by Cramer's rule

$2x - y = 3, \quad x + 2y = 4.$

(d) Find the inverse of the matrix

$$A = \begin{bmatrix} -2 & -1 \\ 1 & -3 \end{bmatrix}.$$

(e) Prove that $2\tan^{-1}\left(\frac{1}{3}\right) + \tan^{-1}\left(\frac{1}{7}\right) = \frac{\pi}{4}$

(*f*) Determine the value of '*a*' so that the points (1, 4), (2, 7), (3, *a*) are collinear.

(*g*) Find the equation of the line passing through (−4, 2) and parallel to the line $4x - 3y = 0$.

(*h*) Find the equation of the plane which passes through the point (1, −1, 4) and is parallel to the plane $2x + 3y + 7z = 11$.

3. Show that

$$\begin{vmatrix} b+c & a & b \\ c+a & c & a \\ a+b & b & c \end{vmatrix} = (a+b+c)(a-c)^2$$ 10

4. If $A + B = 45^\circ$, prove that

(*i*) $(1 + \tan A)(1 + \tan B) = 2$

Hence deduce the value of $\tan 22\frac{1}{2}^\circ$ and $\cot 22\frac{1}{2}^\circ$. 10

5. Find the equation of the line through the point of intersection of $x + 3y - 2 = 0$ and $x - 2y + 4 = 0$ and is perpendicular to the line $2y + 5x + 9 = 0$. 10

6. Show that the points $A(1, 2, 3)$, $B(-1, -2, -1)$, $C(2, 3, 2)$ and $D(4, 7, 6)$ are the vertices of a parallelogram $ABCD$ but it is not a rectangle. 10

7. Find the equation of the sphere passing through the point (1, 2, −3) and (3, −1, 2) and centre lying on *y*-axis. 10